Some familiar topics in Analysis

A student can do a research project in any of the areas listed below. The list below is not exhaustive; students may contact relevant faculty members based on their interests. This may involve lectures in the second year, or a reading course with discussion, as advised by the faculty members.

Inverse Problems Mathematical inverse problems refer to a class of problems where the goal is to determine the unknown causes or parameters of a system based on observed data or measurements. A typical inverse problem is to recover the coefficients of a PDE, i.e. physical properties like conductivity, medium propagation, internal density distribution etc. from the boundary or exterior measurements.

°Some familiar Topics: Carleman estimates, Unique Continuation, Introduction to Calderon problem for Elliptic and Parabolic equation, Scattering problem, Gelfand problem for Wave equation.

°Reference:

1. Isakov, Victor, Inverse problems for partial differential equations. Applied Mathematical Sciences, 127. Springer-Verlag, New York, 1998.
2. Eskin, Gregory, Lectures on linear partial differential equations. Graduate Studies in mathematics, 123. American Mathematical Society, Providence, RI, 2011.
3. Katchalov, Alexander; Kurylev, Yaroslav; Lassas, Matti; Inverse boundary spectral problems. Chapman & Hall/CRC Monographs and Surveys in Pure and Applied Mathematics, 123. Chapman & Hall/CRC, Boca Raton, FL, 2001.

Introduction to the Mathematical Fluid Mechanics Mathematical fluid mechanics deals with the study of fluid flow using mathematical models and techniques. It provides a theoretical framework to understand and analyze the behavior of fluids in various physical systems. This involves PDEs (for example Navier-Stokes equation) which are usually derived from the principles of conservation of mass, momentum, and energy.

°Some familar Topics: Boundary layers, Instability analysis, PDEs as gradient flows, Fluid-structure Interaction problem, Thin film flows.

°Reference:

1. Galdi, G. P. An introduction to the mathematical theory of the Navier-Stokes equations. Steady-state problems. Second edition. Springer Monographs in Mathematics. Springer, New York, 2011.
2. Ockendon, H.; Ockendon, J. R., Viscous flow. Cambridge Texts in Applied Mathematics. Cambridge University Press, Cambridge, 1995.

Homogenization Homogenization processes provide a macro scale approximation to a problem with heterogeneities at micro scale by suitably averaging out small scales and incorporating their effects on large scales. These effects are quantified by the so-called homogenized coefficients. A common thread is often projected in order to come up with their optimal bounds, spectral characterization etc.

°Some familiar Topics: Asymptotic Analysis, H-limit, Compensated Compactness, H-Measure, Relative homogenization.

°Reference:

1. Tartar, Luc, The general theory of homogenization. A personalized introduction. lecture Notes of the Unione Matematica Italiana, 7. Springer-Verlag, Berlin; UMI, Bologna, 2009.
2. Allaire, Gregoire, Shape optimization by the homogenization method. Applied Mathematical Sciences, 146. Springer-Verlag, New York, 2002.

Harmonic Analysis Harmonic analysis is a branch of mathematics concerned with investigating the connections between a function and its representation in frequency. The frequency representation is found by performing Fourier analysis.

°Some familar Topics: Maximal Function, the Riesz-Thorin Interpolation Theorem. Singular integrals, the Calderon-Zygmund Decomposition, Singular integral operators which commute with dilations, vector-valued analogues, Riesz transforms, Poisson integrals, approximate identities, spherical harmonics The Little wood-Paley g-function, multipliers, dyadic decomposition, the Hormander- Mihilin Multiplier Theorem, the Marcinkiewicz Multiplier Theorem.

°Reference:

1. J. Duoandikoetxea, Fourier analysis, Graduate Studies in Mathematics, 29, American Mathematical Society, Providence, RI, 2001.
2. E. M. Stein, Singular integrals and differentiability properties of functions, Princeton Mathematical Series, No. 30, Princeton University Press, Princeton, N.J., 1970.