Mathematics M.Sc. Programme at HRI

Annexure

1 Structural details

We offer one-year and two-year Master's programmes, ***see the details here***. *The M.Sc. course curriculum aims to be more research-oriented, offering alternatives for extra reading courses as early as in the third semester, and with adequate time in the final semester, students are expected to write a master's thesis on that.*

In two years, we offer four semesters with course distribution as follows:

1.1 Year-I: Total Credit 36 (as per UGC norms)

There will be two semesters.

Semester I *(Credit* $(3 \times 5) + 4 = 19$*)*

(1) Algebra I
(2) Topology
(3) Linear Algebra and Calculus on $\mathbb{R}^n$
(4) Measure Theory and Probability

Semester II *(Credit* $(3 \times 4) + 5 = 17$*)*

(1) Algebra II
(2) Functional Analysis
(3) Differential Geometry
(4) Complex Analysis

1.2 Year II: Total Credit: 36 (as per UGC norms)

There will be two semesters.

Semester III *(Credit* $(3 \times 5) + 4 = 19$*)*

(1) Differential Manifolds
(2) Differential Equations
(3) Algebraic Topology
(4) Reading Course / Elective Course

Semester IV *(Credit* $(2 \times 4) + 9 = 17$*)*

(1) Elective Course
(2) Elective Course
(3) Master's Thesis

1.3 Elective Courses for Semester IV

(1) Advanced Functional Analysis and PDEs
(2) Algebraic Topology II
(3) Number Theory
(4) Numerical Methods

1.4 Reading Courses

(*) Any above Elective Course

(1) Introduction to Algebraic Geometry

(2) Lie Algebras

(3) Algebraic Number Theory

(4) Analytic Number Theory

(5) Riemann Surfaces

(6) Advanced Algebraic Geometry

(7) Homological Algebra

(8) Local Fields

(9) Galois Cohomology of elliptic curves

(10) Combinatorial Group Theory

(11) Fourier and Introductory Harmonic Analysis

(12) Representation Theory of Finite Groups

(13) Riemannian Geometry

(14) Discrete Mathematics

- Semester III & IV are made up of three compulsory courses, two **distinct** elective courses, one reading course in Semester III and further continuation of it leading to a master's thesis in Semester IV.

    1. Elective Course:
        - Based on student interest, additional elective courses in special topics in algebra, analysis, topology, geometry might be offered in addition to those on the list above. The syllabus and required readings will be made public well in advance.
    2. Reading Course:
        - Under the supervision of a mentor, a student is required to examine readings (preparatory material) related to the master's thesis. The student is required to present a seminar discussion as part of this course to a group of faculty and graduate students. The talk's topic will be chosen after consulting with the advisor.
        - Furthermore, any elective can be taken as a reading course. In this instance, the student is not permitted to enrol in that course as an elective.
    3. Master's Thesis:
        - Each student is required to complete a dissertation, ideally 35 to 45 pages long, under the direction of a mentor on a topic related to current research.
        - An Inspire fellow, visiting scientist, accomplished post-doctoral fellow, member of the faculty, or both could serve as the mentor. Students will also have the option to choose to complete the master's thesis in another institution, having an MoU with HBNI/HRI. The thesis needs to be defended as per HBNI ordinances.

2 Our offering:

This document is a detailed proposal for a one-year and two-year master's degree in mathematics offered by HRI, compatible with the New Education Policy already put forth by the Central Government of India.

- Students who have completed at-least three years of undergraduate study are eligible to apply for our Master's programme.
  - * After completion of 3 years of undergraduate study they are eligible to apply for our 2 years Master's programme.
    - # After the successful completion of two years, they will get **2 years M.Sc degree** (total credit 72) in Mathematics. It involves reading courses in the second year, which leads to a master's thesis.
    - # However, after the first year of completion, there is an "exit" option with the appropriate certification from the institute. This is a Master's programme that lasts one year (M1). The successful candidates will get **1 year M.Sc. diploma** (total credit 36) in Mathematics.
  - * A student who has completed 4 years of undergraduate study is eligible to enroll in the second year of the proposed two-year Master's programme to study only the advanced courses offered in the second year of the Master's programme. This also entails reading courses that results in a master's thesis. This is an advanced master's programme that lasts one year (M2). The successful candidate will get **1 year M.Sc. degree** (total credit 36) in Mathematics.
- Throughout the academic year, HRI will operate both the (M1) and (M2) programmes concurrently.
- Well-performing students will have the provisions for transitioning from this programme into the HRI Ph.D. programme, if approved by HBNI.
- **Selection Proposal**: There will be two stages to the selection procedure. A written test will follow after the screening, which will involve the national-wide NBHM exam in mathematics. For the NBHM exam, we will set a cutoff score, above which all students will be invited to give the written test. This year-wise threshold may differ for programmes leading to the 2-year M.Sc. degree, 1-year M.Sc. diploma (M1), and 1-year M.Sc. degree (M2).

Detailed Syllabi of the Courses

3 Semester I

3.1 Semester I: Algebra I

1	**Title of the course: Algebra I**
2	**Course Code:** HRI-MM-611-C
3	**Total Contact hours:** Teaching 45 + Tutorial 30
4	**Course Credit:** 5
5	**Course Prerequisites:** Basic set theory, relations and functions.
6	**Assessment (Evaluation Procedure):** Written Examinations 80% + Assignments 20%
7	**Course Outcome:** • To grasp basic ideas of the course through lots of worked-out examples. • To understand tools and techniques to prove theorems and solve problems. • To prepare the student for subsequent courses., e.g., Galois Theory, Commutative algebra, Combinatorial group theory, Representation theory of groups.
8	**Syllabus:** • Groups, subgroups, normal subgroups, Quotient groups, Homomorphism, Isomorphism, Isomorphism theorems, Automorphism. • Normal and subnormal series, The Jordan-Hölder theorem, Group acting on a set, Class equation, Symmetric groups, Cayley's theorem, Sylow's theorems, Applications. Structure theorem for finitely generated Abelian groups. • Central series, Lower and upper central series, Nilpotent groups, Derived series, Solvable and super-solvable groups. • Rings, Subrings, Ideals, Quotient rings, Homomorphisms, Algebra on ideals, Prime, maximal and principle ideals. • Integral domains, Principal integral domains, Unique factorization domains, Euclidian domains. • Polynomial rings, Symmetric polynomials, Noetherian and Artinian conditions, The Hilbert basis theorem. • Modules, Submodules, Homomorphisms, Isomorphism theorems, Direct sum and direct product, Tensor product, Bijective maps and forms, Free modules, Projective and injective modules (basics only).
9	**References:** 1. S. Lang, *Algebra*, Revised third edition, Grad. Texts in Math., 211 Springer-Verlag, New York, 2002. 2. M. Artin, *Algebra*, Prentice Hall, Inc., Englewood Cliffs, NJ, 1991. 3. D.S. Dummit and R.M. Foote, *Abstract algebra*, Third edition John Wiley & Sons, Inc., Hoboken, NJ, 2004. 4. N. Jacobson, *Basic algebra I*, Second edition W. H. Freeman and Company, New York, 1985. 5. N. Jacobson, *Basic algebra II*, Second edition W. H. Freeman and Company, New York, 1989.

3.2 Semester I: Topology

1	**Title of the course:** Topology
2	**Course Code:** HRI-MM-612-C
3	**Total Contact hours:** Teaching 45 + Tutorial 30
4	**Course Credit:** 5
5	**Course Prerequisites:** Undergraduate knowledge of sets and metric spaces.
6	**Assessment (Evaluation Procedure):** Written Examinations 80% + Assignments 20%
7	**Course Outcome:** • Build the foundation of general Topology apply it to understand topological groups and Fundamental groups. • Study basic topological concepts like connectedness, compactness, completeness and their applications to homeomorphisms. Also see interrelationships between various topologies. • Acquire knowledge to pursue further courses/research in differential geometry, Riemannian geometry and algebraic geometry.
8	**Syllabus:** • Revise basics of Topological spaces and properties of continuous maps; Open and closed sets, basis, closure, interior and boundary; Subspace topology, Hausdorff spaces; Pasting Lemma Homeomorphisms. Product topology. • Connected, path-connected and locally connected spaces; Lindelöf and Com- pact spaces, Locally compact spaces, one-point compactification and Tychonoff's theorem; Paracompactness and Partitions of unity (if time permits). • Countability and separation axioms; Urysohn embedding lemma and metrization theorem for second countable spaces. Urysohn's lemma, Tietze extension theorem and applications. Complete metric spaces. Baire Category Theorem and applications. • Quotient topology; Quotient of a space by a subspace; Group action, Orbit spaces under a group action. • Homotopy of maps; Homotopy of paths; Fundamental Group, Seifert–Van Kampen theorem.
9	**References:** 1. J. R. Munkres, *Topology: a first course*, Prentice-Hall (1975). 2. G.F. Simmons, *Introduction to Topology and Modern Analysis*, TataMcGraw- Hill (1963). 3. M.A. Armstrong, *Basic Topology*, Springer (1983). 4. J. L. Kelley, *General Topology*, Springer-Verlag (1975). 5. J. Dugundji, *Topology*, UBS (1999). 6. I. M. Singer and J. A. Thorpe, *Lecture notes on elementary topology and geometry*, UTM, Springer.

3.3 Semester I: Linear Algebra and Calculus on $\mathbb{R}^n$

1	**Title of the course:** Linear Algebra and Analysis on $\mathbb{R}^n$
2	**Course Code:** HRI-MM-613-C
3	**Total Contact hours:** Teaching 45 + Tutorial 30
4	**Course Credit:** 5
5	**Course Prerequisites:** Basic (bachelor level) knowledge in Matrix Algebra, Calculus and Real analysis
6	**Assessment (Evaluation Procedure):** Written Examinations 80% + Assignments 20%
7	**Course Outcome:** • Laying the groundwork for finite and infinite dimensional analysis. • Develop tools and techniques to visualize the structures in Euclidean (flat) spaces. • Several subsequent courses are supported by comprehending this course.
8	**Syllabus:** 1. Linear Algebra • Review of basic Linear Algebra: Matrices, eigenvalues, Cayley-Hamilton Theorem, diagonalizability, vector spaces, linear transformations, rank and nullity. • Triangularization, Schur lemma, S+N decomposition, semi-simple, nilpotent, Jordan canonical forms. • Inner-product spaces, orthogonality, operator norms, spectral radius, normal, Hermitian, unitary operators, spectral Theorem. • Bilinear forms, positive definite operator, square-root of a positive operator, polar decomposition, isometry, rigid motions, the rotation group. 2. Calculus on $\mathbb{R}^n$ • Review of basic analysis in $\mathbb{R}$: Sequences of functions, uniform convergence, Ascoli-Arzela Theorem. • Metric Topology of $\mathbb{R}^n$, Topology induced by l_p norms ($p = 1, 2, \infty$) on $\mathbb{R}^n$ and their equivalence, compactness, connectedness. • Continuity, differentiation - directional derivatives and Fréchet derivatives, Taylor expansion. • Mean value Theorems, maxima, minima, Lagrange multiplier, Hessian matrix, inverse and implicit function Theorems. • Functions of bounded variation, Riemann integration in $\mathbb{R}^n$, change of variable, Jacobian formula.
9	**References:** 1. K. Hoffman and R. Kunze, *Linear Algebra,*, Prentice-Hall of India. 2. P. Lax, *Linear Algebra and its Applications*, Wiley. 3. W. Rudin, *Principles of Mathematical Analysis*, McGraw-Hill. 4. C.C. Pugh, *Real Mathematical Analysis*, Springer. 5. T. Apostol, *Mathematical Analysis*, Narosa; *Calculus*, Wiley Eastern. 6. M. Spivak, *Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus*, Addison-Wesley.

3.4 Semester I: Measure Theory and Probability

1	**Title of the course:** Measure Theory and Probability
2	**Course Code:** HRI-MM-614-C
3	**Total Contact hours:** Teaching 45 + Tutorial 30
4	**Course Credit:** 4
5	**Course Prerequisites:** Real analysis
6	**Assessment (Evaluation Procedure):** Written Examinations 80% + Assignments 20%
7	**Course Outcome:** • Laying the ground work for advanced mathematical analysis. • Exposition of modern tools and techniques to develop finer structures. • Several subsequent courses are supported by comprehending this course.
8	**Syllabus:** 1. Measure theory • Abstract measure theory: σ-algebra, definition and examples of measure spaces, Borel sets, Carathéodory extension, completion of measure spaces. • Construction of Lebesgue and Lebesgue-Stieltjes measures on $\mathbb{R}$. • Measurable functions, Lebesgue integration, convergence theorems: Fatou's lemma, Monotone and Dominated Convergence Theorem. • Product measure, Fubini's Theorem. • Lebesgue decomposition, Radon-Nikodym Theorem. • Absolutely continuous functions, fundamental theorem of calculus. • L^p spaces, duality, Lebesgue differentiation Theorem, convolution and Fourier transform. 2. Probability • Probability spaces, distributions, random variables, expectation, Borel-Cantelli lemma. • Inequalities: Hölder, Cauchy-Schwarz, Jensen, Markov, Chebyshev, Chernoff, Janson. • Convergence notions: convergence in probability and almost sure, L^p. • Central limit Theorem.
9	**References:** 1. H.L. Royden, *Real Analysis*, Macmillan Publishing Company. 2. W. Rudin, *Real and Complex Analysis*, McGraw Hill. 3. G.B. Folland, *Real Analysis: Modern Techniques and Their Applications*, Wiley. 4. R.B. Ash and C.A. Doleans-Dade, *Probability and measure theory*, Academic Press. 5. P. Billingsley, *Probability and measure*, Wiley.

4 Semester II

4.1 Semester II: Algebra II

1	**Title of the course:** Algebra II
2	**Course Code:** HRI-MM-621-C
3	**Total Contact hours:** Teaching 45 + Tutorial 30
4	**Course Credit:** 5
5	**Course Prerequisites:** Group Theory, Properties of commutative rings with identity, integral domains, principal ideal domains, prime ideals and maximal ideals.
6	**Assessment (Evaluation Procedure):** Written Examinations 80% + Assignments 20%
7	**Course Outcome:** • Understand polynomial rings, irreducible polynomials and criteria for irreducibility. • Learn field theory, field extensions and algebraic closure of fields, commutative rings and modules over commutative rings. • Acquire an understanding to pursue further courses/research in commutative algebra, algebraic number theory and algebraic geometry.
8	**Syllabus:** • Polynomial rings; factorial rings and basic properties; irreducible polynomials; criteria for irreducibility; Hilbert's theorem. Algebraic extensions; finite extensions; algebraic closure of a field; splitting field of a polynomial; separable extensions; primitive element theorem. Finite fields; transcendental extensions; inseparable extensions. • Finite Galois extensions; Galois group; Fundamental theorem of Galois theory; examples of finite Galois extensions; quadratic extensions, cubic extensions and biquadratic extensions; Cyclotomic extensions and Abelian extensions over $\mathbb{Q}$. Insolvability of the quintic; Constructible numbers; Norm and trace of finite extensions; Cyclic extensions and Hilbert's theorem 90, Normal basis theorem, Artin-Schreier theorem. • Finitely generated modules, Nakayama's lemma; Tensor product of modules and algebras. Localization; Extension and contraction of ideals in rings of fractions; Primary ideals and primary decomposition theorem. • Integral extension of rings; Going up theorem; Going down theorem. Chain conditions; Noetherian rings; Primary decomposition in Noetherian rings; Hilbert Nullstellensatz. Artin rings and structure theorem for Artin rings.
9	**References:** 1. M. F. Atiyah and I. G. Macdonald, *Introduction to Commutative Algebra*, Addison-Wesley Ser. Math. Westview Press, Boulder, CO, 2016. 2. D. Eisenbud, *Commutative algebra with a view toward algebraic geometry*, Grad. Texts in Math., 150 Springer-Verlag, New York, 1995. 3. D.S. Dummit and R.M. Foote, *Abstract algebra*, Third edition John Wiley & Sons, Inc., Hoboken, NJ, 2004. 4. S. Lang, *Algebra*, Revised third edition, Grad. Texts in Math., 211 Springer-Verlag, New York, 2002. 5. M.P. Murthy et. al., *Galois Theory*, TIFR Pamphlet No. 3.

4.2 Semester II: Functional Analysis

1	**Title of the course:** Functional Analysis
2	**Course Code:** HRI-MM-623-C
3	**Total Contact hours:** Teaching 45 + Tutorial 30
4	**Course Credit:** 4
5	**Course Prerequisites:** Measure theory and Linear algebra
6	**Assessment (Evaluation Procedure):** Written Examinations 80% + Assignments 20%
7	**Course Outcome:** • Exposition of infinite dimensional calculus. • Introductory to modern analysis arising naturally and frequently in mathematics and physics, e.g. Hilbert space.
8	**Syllabus:** • Topological vector spaces, examples, properties, locally convexity, metrizability. • Normed linear spaces and Banach spaces. Bounded linear operators. Dual space. • Hahn-Banach Theorem, applications of Baire Category Theorem: open mapping, closed graph and uniform boundedness theorems. • Weak, weak* topologies, Banach-Alaoglu Theorem, reflexivity. • L^p-spaces and their duality, Stone-Weierstrass Theorem. • Hilbert spaces, adjoint operators, self-adjoint, normal, unitary operators. • Spectrum, spectral radius, analysis of the spectrum of a compact operator. • Complex measure, Riesz representation theorem for the space $C(X)$.
9	**References:** 1. W. Rudin, *Functional analysis*, McGraw-Hill. 2. J.B. Conway, *A course in functional analysis*, Springer. 3. K. Yosida, *Functional analysis*, Springer. 4. S. Kesavan, *Functional analysis*, Hindustan Book Agency.

4.3 Semester II: Complex Analysis

<table>
<tr><td>1</td><td>Title of the course: Complex Analysis</td></tr>
<tr><td>2</td><td>Course Code: HRI-MM-623-C</td></tr>
<tr><td>3</td><td>Total Contact hours: Teaching 45 + Tutorial 30</td></tr>
<tr><td>4</td><td>Course Credit: 5</td></tr>
<tr><td>5</td><td>Course Prerequisites: Real analysis and basics of complex numbers</td></tr>
<tr><td>6</td><td>Assessment (Evaluation Procedure): Written Examinations 80% + Assignments 20%</td></tr>
<tr><td>7</td><td>Course Outcome:
Investigating one of the classical branch of mathematics in modern time.
Learning special tools and techniques to discover complex structures and limitations, e.g. a differentiable function of a complex variable is equal to its Taylor series.</td></tr>
<tr><td>8</td><td>Syllabus:
Complex numbers, topology of $\mathbb{C}$, uniform convergence, Arzela-Ascoli Theorem.
Complex differentiation, power series, analytic function, Cauchy-Riemann equation, holomorphic functions.
Complex integration, index of a closed curve, Cauchy's Theorem and Cauchy's integral formula, zeros of analytic functions, Rouche's . theorem, open mapping theorem, Goursat's Theorem.
Complex logarithm, singularities, classification of singularities, residues and its application on finding definite integrals.
Meromorphic functions, Casorati-Weierstrass Theorem, Rouché's Theorem.
Maximum Modulus Principle, Liouville's Theorem, Morera's Theorem.
Möbius transform, conformal maps, Schwartz Lemma, automorphisms of a disc.
Compactness and Convergence in the complex function space: spaces of analytic functions, meromorphic functions, Riemann mapping Theorem, Weierstrass factorization Theorem, Gamma function, Riemann zeta function.
Runge Theorem, Mittag-Leffler Theorem.
Range of analytic functions: Little Picard Theorem, Great Picard Theorem.</td></tr>
<tr><td>9</td><td>References:
L. Ahlfors, Complex Analysis.
J.B. Conway, Functions of one complex variable, Springer.
E.M. Stein, R. Shakarchi, Complex Analysis. Princeton Lecture Notes.
T.W. Gamelin, Complex Analysis, Springer.</td></tr>
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4.4 Semester II: Differential Geometry

<table>
<tr><td>1</td><td>Title of the course: Differential Geometry</td></tr>
<tr><td>2</td><td>Course Code: HRI-MM-624-C</td></tr>
<tr><td>3</td><td>Total Contact hours: Teaching 45 + Tutorial 30</td></tr>
<tr><td>4</td><td>Course Credit: 4</td></tr>
<tr><td>5</td><td>Course Prerequisites: Topology, Real Analysis.</td></tr>
<tr><td>6</td><td>Assessment (Evaluation Procedure):</td></tr>
<tr><td>7</td><td>Course Outcome:
Understanding basic notions and fundamentals of Differential Geometry.
Acquire knowledge to study courses like Differential Manifolds, Riemannian Geometry etc.</td></tr>
<tr><td>8</td><td>Syllabus:
Review of $\mathbb{R}^n$ calculus, multivariable differentiation.
Inverse function theorem, implicit function theorem and its applications.
Integral calculus, line/surface Integrals, Green/divergence/Stokes theorem, differential forms.
Curves (in plane and space) - local properties, Curves - global properties.
Surfaces, surface of revolution, First fundamental form; Curvature, Second fundamental form.
Gaussian curvature, mean curvature; examples, classifications and computations; Gauss map.
Geodesics; basic properties, geodesic equation, shortest path.
flat surfaces, minimal surfaces, compact surfaces, isometry of surfaces.
Gauss's theorem (Theorema Egregium), Gauss-Bonnet theorem; application on compact surfaces.</td></tr>
<tr><td>9</td><td>References:
M. do Carmo Differential Geometry of Curves and Surfaces, Prentice-Hall
A. Pressley, Elementary Differential Geometry, Springer.
T. J. Willmore An Introduction to Differential Geometry, Dover publication.
M. Spivak, Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus, Addison-Wesley.
M. Spivak, A Comprehensive Introduction to Differential Geometry, Vol. I-V. Publish or Perish, Inc.</td></tr>
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5 Semester III

5.1 Semester III: Differential Manifolds

<table>
<tr><td>1</td><td>Title of the course: Differential Manifolds</td></tr>
<tr><td>2</td><td>Course Code: HRI-MM-631-C</td></tr>
<tr><td>3</td><td>Total Contact hours: Teaching 45 + Tutorial 30</td></tr>
<tr><td>4</td><td>Course Credit: 5</td></tr>
<tr><td>5</td><td>Course Prerequisites: Topology, Differential Geometry, Linear algebra and Real analysis.</td></tr>
<tr><td>6</td><td>Assessment (Evaluation Procedure):</td></tr>
<tr><td>7</td><td>Course Outcome:
• Understand the abstract notion of Differentiable manifolds, differential forms, integration on manifolds.
• Acquire knowledge to pursue further studies in Differential geometry and related topics.</td></tr>
<tr><td>8</td><td>Syllabus:
1. Differentiable manifolds
• Basic notions; the effects of second countability and Hausdorffness, smooth maps.
• Tangent and cotangent spaces, sub-manifolds, consequences of the Inverse Function Theorem,
• Vector fields and their flows, Frobenius Theorem; Sard's theorem.
2. Differential forms
• Recapitulation of multi-linear algebra, tensors, differential forms.
• The de Rham complex and its behavior under differentiable maps, the Lie derivative.
3. Integration on manifolds
• Orientation, the integral of differential forms on differentiable singular chains, integration of differential forms of top degree on an oriented differentiable manifold,
• The theorems of Stokes, the volume form on an oriented Riemannian manifold, the divergence theorem, integration on a Lie group.
4. de Rham cohomology
• Definition, real differentiable singular cohomology, statement of the de Rham theorem, the Poincare lemma.</td></tr>
<tr><td>9</td><td>References:
1. John M. Lee, Introduction to smooth manifolds, Springer New York
2. F. W. Wamer, Foundations of differentiable manifolds and Lie groups, Springer.
3. I. Madsen and J. Tomehave, From Calculus to Cohomology: De Rham Cohomology and Characteristic Classes, Cambridge University Press.</td></tr>
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5.2 Semester III: Differential Equations

<table>
<tr><td>1</td><td>Title of the course: Differential Equations</td></tr>
<tr><td>2</td><td>Course Code: HRI-MM-632-C</td></tr>
<tr><td>3</td><td>Total Contact hours: Teaching 45 + Tutorial 30</td></tr>
<tr><td>4</td><td>Course Credit: 5</td></tr>
<tr><td>5</td><td>Course Prerequisites: Real analysis, linear algebra, Calculus on $\mathbb{R}^n$.</td></tr>
<tr><td>6</td><td>Assessment (Evaluation Procedure): Written Examinations 80% + Assignments 20%</td></tr>
<tr><td>7</td><td>Course Outcome:
• Exposition to analysis and applications in mathematical physics through differential equations
• Acquire an understanding of analysis to do further research in related fields.</td></tr>
<tr><td>8</td><td>Syllabus:
1. Ordinary Differential Equations
• Review of ODE: 1st and 2nd order equations, power series, variation of parameters and Wronskian.
• Existence, uniqueness of initial value problems of 1st order non-linear ODE: Picard's and Peano's Theorems, Gronwall inequality, continuation of solutions, maximal interval of existence, continuous dependence.
• Linear systems: matrix exponential solution, non-homogeneous equation, higher order systems.
• Autonomous systems and phase space analysis: flow, critical points, proper (sink) and improper (source) nodes, spiral points and saddle points.
• Asymptotic behavior: linearized stability, Lyapunov methods.
2. Partial Differential Equations
• First order PDE, solutions by characteristics.
• Classification of 2nd order PDEs, Laplace, Heat and Wave equations, fundamental solutions and Green functions, mean value property, maximum principles, uniqueness of solutions.
• Solutions by other methods: separation of variables, similarity methods, transform methods, power series method, Cauchy- Kowalewsky Theorem, Holmgren's uniqueness Theorem.</td></tr>
<tr><td>9</td><td>References:
1. M.W. Hirsch and S. Smale, Differential Equations, Dynamical Systems, and Linear Algebra, Academic Press.
2. L. Perko, Differential Equations and Dynamical Systems, Springer.
3. L.C. Evans, Partial Differential Equations, American Mathematical Society.
4. F. John, Partial Differential Equations, Springer.
5. R.C. McOwen, Partial Differential Equations: Methods and Applications, Pearson.</td></tr>
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5.3 Semester III: Algebraic Topology

<table>
<tr><td>1</td><td>Title of the course: Algebraic Topology</td></tr>
<tr><td>2</td><td>Course Code: HRI-MM-633-C</td></tr>
<tr><td>3</td><td>Total Contact hours: Teaching 45 + Tutorial 30</td></tr>
<tr><td>4</td><td>Course Credit: 4</td></tr>
<tr><td>5</td><td>Course Prerequisites: Topology, Real Analysis.</td></tr>
<tr><td>6</td><td>Assessment (Evaluation Procedure):</td></tr>
<tr><td>7</td><td>Course Outcome:
Understanding basic notions and fundamentals of algebraic topology.
Acquire knowledge to pursue further topics in algebraic geometry, geometric Groups theory etc.</td></tr>
<tr><td>8</td><td>Syllabus:
Categories; functors and natural transformations. Review the notions of homotopy and the definition of the fundamental group of a topological space.
Free products of groups; Seifert–Van Kampen theorem. Covering spaces; lifting criterion; existence of universal coverings; classifications of covering; automorphism of coverings.
The Eilenberg-Steenrod Axioms, Δ-complexes, simplicial homology, singular homology, the Mayer-Vietoris sequence, The Jordan-Brouwer Separation Theorem.
The Universal coefficient theorem, the Kunneth formula, CW complexes, cellular homology, Lefschetz fixed point theorem.</td></tr>
<tr><td>9</td><td>References:
A. Hatcher, Algebraic Topology, Cambridge University Press.
J. P. May, A Concise Course in Algebraic Topology, University of Chicago Press
J. Munkres, Elements Of Algebraic Topology, CRC Press.
M. J. Greenberg and J. R. Harper, Algebraic topology : A first course.</td></tr>
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6 Elective Courses

6.1 Elective 1: Advanced Functional Analysis and PDEs

1	**Title of the course:** Advanced Functional Analysis and Partial Differential Equations
2	**Course Code:** HRI-MM-641-E
3	**Total Contact hours:** Teaching 45 + Tutorial 30
4	**Course Credit:** 4
5	**Course Prerequisites:** Measure theory and Functional analysis
6	**Assessment (Evaluation Procedure):** Written Examinations 80% + Assignments 20%
7	**Course Outcome:** • Acquiring knowledge about generalized functions and applications in mathematical physics. • Exposition to advance analysis to do further research in PDE and related fields.
8	**Syllabus:** • Distribution theory, Sobolev Spaces. • Poncaré lemma, Embedding theorems, Rellich's lemma, trace theorems. • Second order elliptic equations, weak formulation, calculus of variation, Lax-Milgram Lemma, stability estimate. • Linear evolution equations, existence of weak solutions, energy methods, semigroup theory. • Fluid equations, existence, regularity.
9	**References:** 1. H. Brezis, *Functional Analysis, Sobolev Spaces, and PDE*, Springer. 2. S. Kesavan, *Topics in Functional Analysis and Application*, New Age International Limited. 3. L.C. Evans, *Partial differential equations*, American Mathematical Society. 4. D. Gilbarg, N.S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer. 5. R. Temam, *Navier-Stokes Equations*, North-Holland Publishing Company. 6. G.P. Galdi, *An Introduction to the Mathematical Theory of the Navier-Stokes Equations*, Springer.

6.2 Elective 2: Algebraic Topology II

1	**Title of the course:** Algebraic Topology II
2	**Course Code:** HRI-MM-643-E
3	**Total Contact hours:** Teaching 45 + Tutorial 30
4	**Course Credit:** 4
5	**Course Prerequisites:** General Topology, Homotopy theory and singular homology.
6	**Assessment (Evaluation Procedure):** Written Examinations 80% + Assignments 20%
7	**Course Outcome:** • Understanding concepts of cohomology and higher homotopy theory. • Acquire an understanding to pursue further courses/research in advanced topology, geometry and homological algebra.
8	**Syllabus:** • Revise basics of singular homology and CW complexes. • Singular cohomology, long exact sequence for cohomology, the Universal Coefficient Theorem. • Cup and cap products; the Kunneth formula; Poincare duality for a topological manifold; • Higher homotopy groups; Whitehead's theorem; Freudenthal suspension theorem; The Hurewicz Theorem. • Fiber bundles and long exact sequence of homotopy groups; Examples of computations of higher homotopy groups.
9	**References:** 1. E. Spanier, *Algebraic topology*, (1995). 2. A. Hatcher, *Algebraic Topology.* 3. G. E. Bredon, *Topology and Geometry*, (1997). 4. J. W Vick, *Homology Theory : An Introduction to Algebraic Topology*, Second edition.

6.3 Elective 3: Number Theory

1	**Title of the course:** Number Theory
2	**Course Code:**HRI-MM-644-E
3	**Total Contact hours:** Teaching 45 and Tutorial 30
4	**Course Credit:** 4
5	**Course Prerequisites:** Basic calculus and logic
6	**Assessment (Evaluation Procedure):** Written Examinations 80%+ Assignments 20%
7	**Course Outcome:** At the conclusion of this course, successful students should be able to: • precisely define mathematical terms relevant to the course. • state and prove many of the fundamental theorems in the basic course in number theory. • perform calculations that will allow them to derive identities. • apply the above-mentioned concepts, techniques and skills in various applications.
8	**Syllabus:** • Divisibility, Linear Diophantine equations, Cardano's method, Congruences, Infinitude of Primes in many congruence classes, Quadratic reciprocity law. • Arithmetic functions and their multiplicativity properties. Introduction to Dirichlet series, Summation formulas: Euler-Maclaurin, Poisson summation etc. Riemann Zeta function. • Chebyshev's theorem for prime numbers, Bertrand's postulate, Prime number theorem and Dirichlet prime number theorem. State Prime k-tuple conjecture and various conjectures related to prime distribution. • Introduction to algebraic numbers and algebraic integers. Cantor's argument for the existence of transcendental numbers. Dirichlet's theorem for rational approximation, Badly approximable numbers. Liouville's theorem and examples of transcendental numbers, State the improvements of Liouville, such as Thue, Siegel, Roth and beyond. • Introduction to combinatorial number theory: Ramsey theorem, Schur's theorem, van der Wardern theorem, Hales-Jewett theorem, Erdos-Ginzburg-Ziv theorem.
9	**References:** 1. G. A. Jones and J. M. Jones, *Elementary Number Theory*, Springer Undergraduate Mathematics Series, 1998. 2. Tom M. Apostol, *Introduction to Analytic Number Theory*, UTM, Springer-Verlag, 1976. 3. G.H. Hardy and E. M. Wright, *An introduction to the theory of numbers*, Oxford University Press, 6th ed., 2008. 4. S. D. Adhikari, *Aspects of Combinatorics and Combinatorial Number Theory*, Narosa Publication House, 2002.

6.4 Elective 4: Numerical Methods

<table>
<tr><td>1</td><td>Title of the course: Numerical Methods</td></tr>
<tr><td>2</td><td>Course Code:HRI-MM-645-E</td></tr>
<tr><td>3</td><td>Total Contact hours: Teaching 45 and Tutorial 30</td></tr>
<tr><td>4</td><td>Course Credit: 4</td></tr>
<tr><td>5</td><td>Course Prerequisites: Analysis, Calculus, Linear Algebra</td></tr>
<tr><td>6</td><td>Assessment (Evaluation Procedure): Written Examinations 60%+ Assignments 40%</td></tr>
<tr><td>7</td><td>Course Outcome:
Developing techniques and skills to do numerics for applied problems.
Getting mathematically trained in the present world of computation and programming.</td></tr>
<tr><td>8</td><td>Syllabus:
Numerical Algorithms and errors: Floating point systems, Roundoff error accumulations.
Interpolation: Lagrange Interpolation, Newton's divided difference interpolation, Finite differences, Hermite Interpolation, Cubic splines, Least square and minimax approximations.
Numerical differentiation and integration: Richardson extrapolation, Trapezoidal rule, Simpson's rule, Newton-Cotes rule, Gauss quadrature rule, Romberg integration rule.
Solutions of Linear Algebraic equations: Direct Methods, Gauss elimination, Pivoting, matrix factorizations.
Iterative Methods: Matrix norms, Jacobi and Gauss-Seidel methods, Relaxation methods.
Computation of Eigenvalues and Eigenvectors: Power Method, QR algorithm, Given's and Householder's methods for symmetric matrices, Rutishauser method for general matrices.
Error analysis and stability of algorithms. Nonlinear equations: Bisection method, Secant method, Newton Raphson method, Muller's method, criterion for acceptance of a root, system of non-linear equations.
Zeros of polynomials, Horner and Muller methods, equations in higher dimensions. Linear system of algebraic equations: Gauss elimination method, LU-decomposition method; matrix inversion, iterative methods, ill-conditioned systems.
Numerical methods for solving IVPs for ODEs: Difference equations, Routh-Hurwitz criterion, Single step methods: Taylor series method, explicit Runge-Kutta methods, convergence, order, relative and absolute stability. Multi-step methods: Adams-Bashforth, Adams-Moulton methods, Systems of ODEs. Boundary value problems, shooting methods, finite differences, Rayleigh-Ritz methods.</td></tr>
<tr><td>9</td><td>References:
K.E. Atkinson, An Introduction to Numerical Analysis, Wiley, 1989.
S. D. Conte and C. De Boor, Elementary Numerical Analysis: An Algorithmic Approach, McGraw-Hill, 1981.
G.H. Golub and J.M. Ortega, Scientific Computing and Differential Equations: An Introduction to Numerical Methods, Academic Press, 1992.</td></tr>
</table>

7 Reading Courses

7.1 Reading 1: Introduction to Algebraic Geometry

<table>
<tr><td>1</td><td>Title of the course:Introduction to algebraic geometry</td></tr>
<tr><td>2</td><td>Course Code: HRI-MM-641-R</td></tr>
<tr><td>3</td><td>Total Contact hours: At least 75 hours of interaction with the instructor</td></tr>
<tr><td>4</td><td>Course Credit: 4</td></tr>
<tr><td>5</td><td>Course Prerequisites: Commutative algebra (Algebra II) and basic topology.</td></tr>
<tr><td>6</td><td>Assessment (Evaluation Procedure): Oral Examination 80% + Presentation 20%</td></tr>
<tr><td>7</td><td>Course Outcome:
Understanding of the algebraic techniques to solve geometric problems.
Computations with concrete varieties and morphisms.
Algebraic analogues of the classical study of complex manifolds.
Acquire an understanding to pursue further courses/research in arithmetic and algebraic geometry.</td></tr>
<tr><td>8</td><td>Syllabus:
Affine algebraic sets, Hilbert Nullstellensatz, Function field, Krull dimension
Graded rings, (quasi-)projective Varieties, rational functions
Algebraic plane curves, morphism, product, finite maps
Normality, Noether normalisation, Birational morphism
Tangent spaces, non-singularity, resolution of singularities, blowing up points
Divisors, Bezut's theorem on curves, Class group
Intersection numbers, Riemann-Roch theorem</td></tr>
<tr><td>9</td><td>References:
I. Shafarevich, Basic Algebraic Geometry I, (Second Edition) Springer-Verlag (1988).
M. Reid, Undergraduate Algebraic Geometry, Cambridge University Press (1988).
W. Fulton, Algebraic Curves, Addison-Wesley Publishing Company, Advanced Book Program (1969).
U. Görtz, T. Wedhorn, Algebraic Geometry I, Vieweg+Teubner Verlag (2010).</td></tr>
</table>

7.2 Reading 2: Lie Algebras

1	**Title of the course:** Lie Algebras
2	**Course Code:** HRI-MM-642-R
3	**Total Contact hours:** At least 75 hours of interaction with the instructor
4	**Course Credit:** 4
5	**Course Prerequisites:** Linear Algebra
6	**Assessment (Evaluation Procedure):** Oral examination 80% + Presentation 20%
7	**Course Outcome:** At the end of the course, successful students will be able to : • Understand basic terms and theorems useful in Lie algebras. • Understand the topic by working out examples of a variety of special types of Lie algebras. • Read independently and continue further studies or research in Lie algebras and representation theory.
8	**Syllabus:** • Lie Groups, examples of Lie groups, Lie algebra of a Lie group, exponential map, closed linear groups and corresponding Lie algebras. • Linear Lie algebras, examples, A_l, B_l, C_l, D_l, Abstract Lie algebras. • Ideals, Homomorphisms and representations of Lie algebras, Automorphisms, Solvable and nilpotent Lie algebras, Engel's theorem. • Lie's theorem for solvable Lie algebras, Jordan-Chevalley decomposition, Cartan's criterion, Killing form, Semisimple Lie algebras, Inner derivations, Abstract Jordan decomposition, Complete reducibility of representations. • Casimir element of a representation, Weyl's theorem, Preservation of Jordan decomposition, Representations of $\mathfrak{sl}(2, F)$, Root space decomposition for a semisimple Lie algebra, centralizer of maximal toral subalgebra, Orthogonality properties, Integrality and rationality properties.
9	**References:** 1. James E. Humphreys, *Introduction to Lie algebras and representation theory*, Graduate Texts in Mathematics, Vol. 9. Springer-Verlag, New York-Berlin, 1972. 2. Anthony W. Knapp, *Lie groups, Lie algebras, and cohomology*, Math. Notes, 34 Princeton University Press, Princeton, NJ, 1988. 3. J.P. Serre, *Lie algebras and Lie groups*, 1964 lectures given at Harvard University. Second edition Lecture Notes in Math., 1500 Springer-Verlag, Berlin, 1992. 4. Karin Erdmann and Mark J. Wildon, *Introduction to Lie algebras*, Springer Undergrad. Math. Ser. Springer-Verlag London, Ltd., London, 2006.

7.3 Reading 3: Algebraic Number Theory

1	**Title of the course:** Algebraic Number Theory
2	**Course Code:** HRI-MM-643-R
3	**Total Contact hours:** At least 75 hours of interaction with the instructor
4	**Course Credit:** 4
5	**Course Prerequisites:** Introduction to Groups, Rings and Modules, Galois theory.
6	**Assessment (Evaluation Procedure):** Oral Examination 80%+ Presentation 20%
7	**Course Outcome:** • Understanding the ring theoretic properties beyond the ring $\mathbb{Z}$, failure of unique factorization in the ring of integers of algebraic number fields and the measure of the failure by understanding the ideal class groups. • Understanding the unique factorization in the ideal level of the ring of integers and a class of examples of Dedekind domains. • Understanding the behavior of prime numbers in the algebraic number fields, Frobenius element of the Galois extensions over $\mathbb{Q}$ captures all the arithmetic properties of integers.
8	**Syllabus:** • Algebraic numbers and algebraic integers, Norm and trace, Algebraic number fields, Integral bases, Monogenety of algebraic number fields, Discriminant of algebraic number fields. Quadratic number fields, Cyclotomic number fields. • Divisibility, UFD, PID, Euclidean domain in algebraic number fields. • Ideal divisors, Fractional ideals, Dedekind domain, Ideal class Groups. Finiteness of ideal class groups. • Units in algebraic number fields, Kronecker's theorem, Minkowski bound, Dirichlet's unit theorem, Cyclotomic units, Algebraic integers lying on the unit circle. • Splitting of rational primes in algebraic number fields, Dedekind criterion for ramification. Kummer-Dedekind theorem on splitting of rational primes, Ramification and discriminant. Decomposition group and inertia groups in a Galois extension over $\mathbb{Q}$, Frobeinus element, Dedekind theorem. State the density theorems of Frobenius and Cebotarev, some of their applications. • Absolute values and completions of number fields.
9	**References:** 1. I. Stewart and D. Tall, *Algebraic Number Theory and Fermat's Last Theorem*, Chapman and Hall/CRC, 2015. 2. A. Frohlich and M. J. Taylor, *Algebraic Number Theory*, Cambridge University Press, 1998. 3. P. Pollack, *Conversational Introduction to Algebraic number theory*, American Mathematical Society, 2019. 4. G. J. Janusz, *Algebraic Number Fields*, American Mathematical Society, 1996. 5. D. A. Marcus, *Number Fields*, Universitext, Springer, 2018.

7.4 Reading 4: Analytic Number Theory

1	**Title of the course:** Analytic Number Theory
2	**Course Code:** HRI-MM-644-R
3	**Total Contact hours:** At least 75 hours of interaction with the instructor
4	**Course Credit:** 5
5	**Course Prerequisites:** Calculus, Real Analysis and Complex Analysis
6	**Assessment (Evaluation Procedure):** Oral Examination 80%+ Presentation 20%
7	**Course Outcome:** At the conclusion of this course, successful students should be able to: • precisely define mathematical terms relevant to the course. • state and prove many of the fundamental theorems in the analytic theory of numbers. • perform calculations that will allow them to derive identities. • apply the above-mentioned concepts, techniques and skills in various applications.
8	**Syllabus:** • Introduction to arithmetic functions and Dirichlet series, Summation formulas: Euler-Maclaurin, Poisson summation etc. • Riemann Zeta function, Functional equation and Analytic continuation, zeros of Riemann Zeta function. • Non-vanishing of Riemann Zeta function at $\Re(s) = 1$ and Prime number theorem. • Dirichlet characters and Gauss sums, Dirichlet L-function, Functional equation and zeros, Primes in arithmetic progressions. • Elementary sieve methods, Bilinear forms and the large sieve. • Introduction to holomorphic modular forms.
9	**References:** 1. H. Iwaniec and E. Kowalski, *Analytic Number Theory*, Colloquium Publications, **53**, American Mathematical Society, 2004. 2. J. P. Serre, *A Course in Arithmetic*, GTM, **7**, Springer-Verlog, 1973. 3. M. Ram Murty, *Problems in Analytic Number Theory*, GTM, **206**, Springer-Verlog, 2008. 4. M. Ram Murty, M. Dewar and H. Graves, *Problems in the theory of Modular Forms*, IMSC Lecture notes series -1, HBA, Delhi, 2015.

7.5 Reading 5: Riemann Surfaces

<table>
<tr><td>1</td><td>Title of the course: Riemann Surfaces</td></tr>
<tr><td>2</td><td>Course Code: HRI-MM-645-R</td></tr>
<tr><td>3</td><td>Total Contact hours: At least 75 hours of interaction with the instructor</td></tr>
<tr><td>4</td><td>Course Credit: 4</td></tr>
<tr><td>5</td><td>Course Prerequisites: Basic knowledge of topology, complex analysis and knowledge of rings and modules.</td></tr>
<tr><td>6</td><td>Assessment (Evaluation Procedure): Oral Examinations 80% + Presentation 20%</td></tr>
<tr><td>7</td><td>Course Outcome:
Understand the basics of complex analytic geometry.
Learn the basics of sheaves and algebraic curves over complex numbers.
To prepare the student for further courses in algebraic geometry, advanced topics in complex analysis and complex analytic geometry.</td></tr>
<tr><td>8</td><td>Syllabus:
Definition of complex atlases, complex charts, Riemann surfaces and complex structure on a manifold. Examples of Riemann surfaces; projective line; complex torus; smooth projective plane curves.
Holomorphic functions on a Riemann surface; meromorphic functions on a Riemann surface with examples. Holomorphic maps between Riemann surfaces; automorphism groups; degree of holomorphic maps; Euler number for compact Riemann surfaces and Hurwitz formula.
Recall basics of covering maps, fundamental groups, group actions on manifolds and quotients. Finite group actions on Riemann surfaces; Hurwitz's theorem; Monodromy of covering and holomorphic maps; monodromy representation.
Differential and holomorphic forms; sheaves; vector bundles, line bundles and divisors. Linear equivalence and forms associated to divisors; finiteness theorems for cohomology on a compact Riemann surface; Dolbeault isomorphism; Weyl's lemma on regularity of $\overline{\partial}$ and Serre duality;
Rieman-Roch theorem; some applications of the Riemann-Roch theorem; Able-Jacobi map and Abel's theorem.</td></tr>
<tr><td>9</td><td>References:
Rick Miranda, Algebraic curves and Riemann surfaces, Grad. Stud. Math., 5 American Mathematical Society, Providence, RI, 1995.
Raghavan Narasimhan, Compact Riemann surfaces, Lectures Math. ETH Zürich Birkhäuser Verlag, Basel, 1992.
M. S. Narasimhan, R. R. Simha Raghavan Narasimhan, C. S. Seshadri, Riemann Surfaces, TIFR Pamphlet.
Otto Forster, Lectures on Riemann surfaces, Translated from the German by Bruce Gilligan Graduate Texts in Mathematics, 81, Springer-Verlag, New York-Berlin, 1981.</td></tr>
</table>

7.6 Reading 6: Advanced Algebraic Geometry (Scheme theory)

1	**Title of the course:** Advanced Algebraic Geometry (Scheme theory)
2	**Course Code:** HRI-MM-646-R
3	**Total Contact hours:** At least 75 hours of interaction with the instructor
4	**Course Credit:** 4
5	**Course Prerequisites**: Commutative Algebra (Algebra II), Category theory
6	**Assessment (Evaluation Procedure):** Oral Examination 80% + Presentation 20%
7	**Course Outcome:** • Understand and learn the advanced topics in Algebraic Geometry. • Learn applications of abstract techniques of scheme theory in geometric and number theoretic problems. • Pursue further courses/research in arithmetic and algebraic geometry.
8	**Syllabus:** • (Pre-)sheaf, Sheaves of modules over topological spaces • Spec of a ring, stalk, co-ordinate ring, dimension • Scheme, Gluing, reduced and integral scheme • Product, separated and proper scheme • Projective morphism, flat morphism, birational map • Coherent sheaves and vector bundles, divisors and line bundles • Sheaf cohomology, derived push-forward and pull-back, base change • Serre duality theorem, semicontinuity theorems • functor of points, Group scheme, Hilbert scheme
9	**References:** 1. I. Shafarevich, *Basic Algebraic Geometry II*, (Second Edition) Springer-Verlag (1988). 2. R. Hartshorne, *Algebraic Geometry*, Graduate texts in Mathematics (1977). 3. D. Mumford and T. Oda, *Algebraic Geometry II*, TRIM publications 73 (2015). 4. Q. Liu, *Algebraic Geometry and Arithmetic Curves*, OUP Oxford (2002). 5. D Eisenbud, J. Harris, *The Geometry of Schemes*, Graduate texts in Mathematics (2000), 6. U. Görtz, T. Wedhorn, *Algebraic Geometry I*, Vieweg+Teubner Verlag (2010).

7.7 Reading 7: Homological Algebra

<table>
<tr><td>1</td><td>Title of the course: Homological algebra</td></tr>
<tr><td>2</td><td>Course Code: HRI-MM-647-R</td></tr>
<tr><td>3</td><td>Total Contact hours: At least 75 hours of interaction with the instructor</td></tr>
<tr><td>4</td><td>Course Credit: 4</td></tr>
<tr><td>5</td><td>Course Prerequisites: Familiarity with rings, modules, projective and injective modules.</td></tr>
<tr><td>6</td><td>Assessment (Evaluation Procedure): Oral Examination 80% + Presentation 20%</td></tr>
<tr><td>7</td><td>Course Outcome:
Gain familiarity with working with categories and functors.
Acquire knowledge of derived categories, derived functors and spectral sequences.
Acquire the necessary background in categories to pursue different topics in mathematics that use a categorical framework.</td></tr>
<tr><td>8</td><td>Syllabus:
Categories; Limits and colimits; Functors and natural transformations; Adjoint functors.
Additive categories and Abelian categories; Chain complexes in an Abelian category; Chain homotopy; Mapping cone and mapping cylinders. Delta functors; Exact functors.
Injective resolutions; Projective resolutions; Cartan-Eilenberg resolutions; Truncation of complexes; EXT and TOR; Double complex and totalization; Spectral sequences and convergence; Leray-Serre spectral sequence; Spectral sequence associated to a filtration and double complex; Grothendieck spectral sequences; spectral sequence associated to an exact couple.
Simplicial objects in a category; Simplicial homotopy groups; Dold-Kan correspondence; Eilenberg-Zilber theorem.
Homotopy category of chain complexes; Triangulated categories; Localization; Derived categories; Left and right exact functors; Derived functors.</td></tr>
<tr><td>9</td><td>References:
Charles A. Weibel, An introduction to homological algebra, Cambridge Stud. Adv. Math., 38 Cambridge University Press, Cambridge, 1994. xiv+450 pp.
H. Cartan and S. Eilenberg, Homological algebra, Princeton University Press, Princeton, N. J., 1956.
J. J. Rotman, An introduction to homological algebra, Second edition Universitext Springer, New York, 2009.
S. I. Gelfand and Y. I. Manin, Methods of homological algebra, Second edition Springer Monogr. Math. Springer-Verlag, Berlin, 2003.
P.J. Hilton and U. Stammbach, A course in homological algebra, Graduate Texts in Mathematics, Vol. 4. Springer-Verlag, New York-Berlin, 1971.</td></tr>
</table>

7.8 Reading 8: Local Fields

1	**Title of the course:** Local fields
2	**Course Code:**HRI-MM-648-R
3	**Total Contact hours:** At least 75 hours of interaction with the instructor
4	**Course Credit:** 4
5	**Course Prerequisites:** General topology, Groups, Rings and Galois theory. A course in algebraic number theory is not necessary but will certainly help.
6	**Assessment (Evaluation Procedure):** Oral Examination 80% + Presentation 20%
7	**Course Outcome:** This course is a gentle introduction to the beautiful theory of local fields. After this foundational course, students will be able to • understand the foundations of p-adic local fields. • acquire knowledge of appropriate results to do analysis on p-adic local fields. • study further research topics on p-adic Galois representation theory and (φ, Γ)-modules.
8	**Syllabus:** • Non-archimedean absolute values and topological properties, Ostrowski's theorem, the field of p-adic numbers $\mathbb{Q}_p$, the ring of p-adic integers $\mathbb{Z}_p$ and its properties, Hensel's lemma, the Teichmüller map, quadratic forms, Hasse-Minkowski theorem. • Elementary analysis in $\mathbb{Q}_p$: Laurent series, continuity, derivatives and convergence conditions of various Laurent series, Strassmann's theorem and its consequences, exponential and logarithm maps and their convergence results. • Extensions of $\mathbb{Q}_p$: extension of absolute values for finite extensions of $\mathbb{Q}_p$, construction and properties of $\mathbb{C}_p$ - the algebraic closure of $\mathbb{Q}_p$, Eisenstein irreducibility criterion over $\mathbb{Q}_p$, the ramification and the residue indices for finite extensions of $\mathbb{Q}_p$, roots of unity in $\mathbb{Q}_p$, the p-th cyclotomic polynomial and its irreducibility, construction of the cyclotomic extension of $\mathbb{Q}_p$, ramified and unramified extensions of local fields, Krasner's lemma, Analysis on $\mathbb{C}_p$, Higher ramification groups and solvability of finite Galois extensions of p-adic fields.
9	**References:** 1. F. Q. Gouvêa, *p-adic numbers*, Springer, 1997. 2. J-P Serre, *Local fields*, Springer, 1979 (Chapters I-V). 3. J.W.S. Cassels and A. Fröhlich, *Algebraic Number Theory*, Academic Press, 1967. 4. Yvette Amice, *Les nombres p-adiques*, Presses Universitaires de France, 1975. 5. G. Bachman, *Introduction to p-adic numbers and valuation theory*, Academic press, 1964.

7.9 Reading 9: Galois Cohomology of elliptic curves

1	**Title of the course:** Galois cohomology of elliptic curves
2	**Course Code:** HRI-MM-649-R
3	**Total Contact hours:** At least 75 hours of interaction with the instructor
4	**Course Credit:** 4
5	**Course Prerequisites:** Topology and Algebraic number theory
6	**Assessment (Evaluation Procedure):** Oral Examination 80% + Presentation 20%
7	**Course Outcome:** This course can be taken in semester IV. After the course, the students will be able to: • understand the foundations of profinite groups and the construction of cohomology groups. • acquire knowledge of the main results in the theory of elliptic curves that will be discussed in class. • acquire an understanding of objects needed to do further research in Iwasawa theory.
8	**Syllabus:** • Profinite groups, cohomology and homology of pro-p groups, restriction, corestriction homomorphisms and functoriality properties, induced modules, cup products, statements of the spectral sequence for group extensions and the inflation-restriction exact sequence, p-cohomological dimension, strict cohomological dimension, interpretations of the dimensions of $H^1(G, \mathbb{Z}/p\mathbb{Z})$ and $H^2(G, \mathbb{Z}/p\mathbb{Z})$ in terms of generators and relations of the pro-p group G. • Elliptic Curves (general overview with statements of main results and examples): Definition of elliptic curves using generalized Weierstrass equations, Tate-module, torsion points, Weil pairing, Hasse theorem for elliptic curves over finite fields, elliptic curves over local fields, definitions of good and bad reduction, elliptic curves over $\mathbb{Q}$, weak Mordell-Weil theorem, Mordell-Weil theorem. • Kummer sequence of elliptic curves via cohomology, definitions of Selmer groups and Shafarevich-Tate groups over number fields, the exact sequence connecting the p-primary Selmer group with the p-part of the Shafarevich-Tate group and the Mordell-Weil group, cyclotomic $\mathbb{Z}_p$-extension of $\mathbb{Q}$, Selmer groups and Shafarevich-Tate groups over the cyclotomic $\mathbb{Z}_p$-extension of $\mathbb{Q}$ and their arithmetic properties.
9	**References:** 1. J-P Serre, *Galois cohomology*, Springer, 1997. 2. L. Washington, *Elliptic Curves: Number Theory and Cryptography*, (second edition), Chapman and hall/CRC, 2008. 3. J. Silverman, *Rational points on Elliptic curves*, Springer, 2015. 4. J. Silverman, *The arithmetic of Elliptic curves*, Springer, 2009. 5. J. Coates and R. Sujatha, *Galois cohomology of elliptic curves*, a publication of Tata Institute of Fundamental Research, 2010.

7.10 Reading 10: Combinatorial Group Theory

1	**Title of the course:** Combinatorial Group Theory
2	**Course Code:**HRI-MM-650-R
3	**Total Contact hours:** At least 75 hours of interaction with the instructor
4	**Course Credit:** 4
5	**Course Prerequisites:** Basic course in group theory, little knowledge of graph theory.
6	**Assessment (Evaluation Procedure):** Oral Examination 80% + Presentation 20%
7	**Course Outcome:** • Understanding presentation of a group, free groups, and their automorphisms. • Learning free product, amalgamation and intricate tricks and techniques in the area. • Acquired knowledge will help a student in pursuing further studies/research in combinatorial group theory, algebraic topology and geometric group theory.
8	**Syllabus:** • Word combinatorics - Construction of a free group on a given set, Definition as a universal object. • Group as a homomorphic image of a free group, Construction of a group using generators and relations. • Elementary properties of free groups, Tietze transformations and Nielsen's method. • Automorphisms of free groups, Cayley graph. • Verbal subgroup, Presentation of a subgroup (The Reidemeister-Schreier method), HNN extensions. • Free products and amalgamation, Embedding theorems, One-relator groups. • Small cancellation theory, Finite quotients (Burnside's problem). • Dehn's fundamental problems and recent developments.
9	**References:** 1. G. Baumslag, *Topics in combinatorial group theory*, Lectures in Mathematics ETH Zurich, Birkhauser Verlag, Basel, 1993. 2. R. C. Lyndon and P. E. Schupp, *Combinatorial Group Theory*, Reprint of the 1977 edition, Springer, 2001. 3. W. Magnus, A. Karrass and D. Solitar, *Combinatorial group theory, Presentations of groups in terms of generators and relations*, Reprint of the 1976 second edition, Dover Publications, 2004. 4. J-P. Serre, *Trees*, Translated from the French by John Stillwell. Springer-Verlag, Berlin-New York, 1980.

7.11 Reading 11: Fourier and Introductory Harmonic Analysis

1	**Title of the course:** Fourier and Introductory Harmonic Analysis
2	**Course Code:** HRI-MM-651-R
3	**Total Contact hours:** At least 75 hours of interaction with the instructor
4	**Course Credit:** 4
5	**Course Prerequisites:** Measure theory and Functional analysis
6	**Assessment (Evaluation Procedure):** Oral Examination 80% + Presentation 20%
7	**Course Outcome:** • Developing mathematical technique that decomposes complex data into simpler (trigonometric) functions, which encompasses a vast spectrum of mathematics. • Acquire an understanding of advanced analysis to do further research in related fields.
8	**Syllabus:** • L^p and weak-L^p space, Riesz-Thorin, Marcinkiewicz Interpolation Theorems. • Maximal functions, Calderón-Zygmund decomposition, Hardy-Littlewood maximal operator. • Fourier series, summability kernels, application in solving PDEs. • Schwartz space, Fourier transform in L^1 and L^p spaces $1 < p \leq 2$, convolution and product formulas, differentiation and polynomial multiplication formulas, Fourier inversion, Plancherel Theorem, Paley-Wiener Theorem. • Tempered distribution, Fourier transform, explicit formulas for Laplace, heat and wave equations via Fourier transform method. • Convolution operators on L^p spaces and multipliers, $M^{p,q}$ spaces, Space of Fourier Multipliers M_p.
9	**References:** 1. E.M. Stein and R. Shakarchi, *Fourier Analysis: An Introduction*, Princeton University Press. 2. Y. Katznelson, *An introduction to harmonic analysis*, Dover Publications. 3. E.M. Stein and G.Weiss, *Introduction to Fourier Analysis on Euclidean Spaces*, Princeton University Press. 4. L. Grafakos, *Classical Fourier Analysis*, Springer.

7.12 Reading 12: Representation Theory of Finite Groups

1	**Title of the course:** Representation Theory of Finite Groups
2	**Course Code:** HRI-MM-652-R
3	**Total Contact hours:** At least 75 hours of interaction with the instructor
4	**Course Credit:** 4
5	**Course Prerequisites:** Basic course on group theory and linear algebra.
6	**Assessment (Evaluation Procedure):** Oral Examination 80% + Presentation 20%
7	**Course Outcome:** • To grasp definitions and important concepts in representation theory for finite groups. • To get handy to the topic by working out examples of a variety of special types of groups. • To enable a student to formulate important results and theorems covered by the course. • Confidently describe the main features of the proofs of important theorems. • To use the theory, methods and techniques of the course to solve mathematical problems.
8	**Syllabus:** • Definition of a group representation and sub-representation, examples. • Irreducible representations and tensor product of two representations. • Associated characters, Schur's lemma, Orthogonality relations, Decomposition theorems. • Induced representations, Compact groups and their representations, Concrete examples. • Group algebras, modules, complete reducibility, Wedderburn's theorem. • Restriction and induction, Mackey's criterion, Representations of super solvable groups. • Artin's Theorem, Brauer's theorem, Applications, Introduction to rationality questions with examples.
9	**References:** 1. G. James, and M. Liebeck, *Representations and characters of groups*, Cambridge Math. Textbooks, Cambridge University Press, Cambridge, 1993. 2. W. Fulton, and J. Harris, *Representation Theory: A First Course*, Graduate texts in mathematics. Vol. 129. New York, NY: Springer, 1991. 3. J.-P. Serre, *Linear Representations of Finite Groups*, Graduate texts in mathematics. Vol. 42. New York, NY: Springer-Verlag, 1977. 4. B. Simons, *Representations of finite and compact groups*, Graduate Studies in Mathematics, 10. American Mathematical Society, Providence, RI, 1996. xii+266 pp.

7.13 Reading 13: Riemannian Geometry

1	**Title of the course:** Riemannian Geometry
2	**Course Code:** HRI-MM-653-E
3	**Total Contact hours:** At least 75 hours of interaction with the instructor
4	**Course Credit:** 4
5	**Course Prerequisites**: Differential Geometry, Linear Algebra and ODE
6	**Assessment (Evaluation Procedure):** Oral Examination 80% + Presentation 20%
7	**Course Outcome:** • Giving the exposure of an important tool in modern mathematics impacting on diverse areas from the pure to the applied. • Our objective is to analyse manifolds with the notions of geodesics and curvature. By the end of the course, students will have a thorough understanding of curved spaces. • An introduction to the applications of Riemannian geometry to topology and geometric problems.
8	**Syllabus:** • Review of Differential Geometry and Differential Manifold: Manifolds and examples, smooth maps, tangent vectors and the tangent bundle, push-forward. • Tensor fields, tensor product. Lie derivative. Partition of unity, orientation, Riemannian metric, Cartan's formula. • Definition of Riemannian manifolds and examples. • Connections and covariant derivatives of vector fields and other tensors. • Levi-Civita connection, parallel transport : The fundamental theorem of Riemannian geometry. • Geodesics, exponential map, curvature and examples. • Geodesic completeness, Hopf-Rinow Theorem. • Riemannian and sectional curvature. • Manifolds with constant curvature, sphere, geometry of hyperbolic space, classification.
9	**References:** 1. John M. Lee, *Riemannian Manifolds: An Introduction to Curvature.* Springer. 2. M. do Carmo, *Riemannian Geometry*, Springer. 3. W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry,* Academic Press. 4. S. Kumaresan, *Riemannian Geometry- Concepts, Examples and Techniques,* Techno World.

7.14 Reading 14: Discrete Mathematics

1	**Title of the course:** Discrete Mathematics
2	**Course Code:** HRI-MM-654-R
3	**Total Contact hours:** At least 75 hours of interaction with the instructor
4	**Course Credit:** 4
5	**Course Prerequisites:** Basic mathematical logic.
6	**Assessment (Evaluation Procedure):** Oral Examination 80%+Presentation 20%
7	**Course Outcome:** • Understanding of quotient groups in the number theoretic settings. • Understanding the Pigeon hole and generalized pigeon hole principle. How to apply this technique to different problems. • Understanding of recurrence relations in various contexts. • Understanding of various ways of counting. Generating functions techniques.
8	**Syllabus:** • **Advanced Counting:** Stirling numbers of the first and second kind, Pigeon hole principle, generalized Pigeon hole principle and its applications, Erdos - Szekere's theorem on monotone subsequences, Ramsey theorem. Theorem of Hilbert, Dirichlet's theorem on rational approximations. Erdos-Gnizburg-Ziv theorem, Inclusion exclusion principle and its applications. Derangements, Permutations with forbidden positions, restricted positions and Rook polynomials. • **Recurrence Relations:** The Fibonacci sequence, linear homogeneous and non-homogeneous recurrence relations. Proof of the solution of linear homogeneous recurrence relations with constant coefficient in case of distinct roots and when they have repeated roots, iteration and induction. Ordinary generating functions, exponential functions for counting combinations with and without repetitions, applications to counting, and the use of generating functions for solving homogeneous and non-homogeneous recurrence relations. Catalan numbers. • **Polya's method of Counting:** Equivalence relations and orbits under the permutation group action. Orbit stabilizer theorem, Burnside lemma and its applications, Cycle index, Polya's formula, Applications of Polya's formula. Theorem of J. P. Serre on Burnside lemma and its applications. • **Basic Graph Theory technique:** Basic graph theory, Handshaking problem, Graphs and matrices, Planar graphs, Hall's marriage theorem, Kneser's theorem.
9	**References:** 1. A. Tucker, *Applied Combinatorics*, John Wiley & Sons, Inc., New York, 1995 2. P. J. Cameron, *Combinatorics: Topics, Techniques, Algorithms*, Cambridge University Press, 1994. 3. S. M. Cioaba and M. Ram Murty, *A first course in Graph theory and Combinatorics*, Texts Read. Math., 55 Hindustan Book Agency, New Delhi Springer, Singapore, 2022.